AERODYNAMIC SUMMARY SHEET 1

ATMOSPHERE

ALT. FT.	PRESSURE PSF	PRESSURE δ	DENSITY SLUG/FT.^{3}	DENSITY σ	TEMP °R
0	2,116.22	1.0000	.002377	1.0000	518.67
10,000	1,455.32	.6877	.001755	.7385	483.01
20,000	972.48	.4595	.001266	.5328	447.35
30,000	628.43	.2970	.000889	.3741	411.69
40,000	391.69	.1851	.000585	.2462	389.97
50,000	242.22	.1145	.000362	.1522	389.97

Sea Level ⟶ 36,089 Ft $\quad A_1 = 6.87535 \times 10^{-6}$

Pressure Ratio $\delta = (1 - A_1 H)^{5.2561}$ $\quad H = 145{,}448\,(1 - \delta^{.19026})$

Density Ratio $\sigma = (1 - A_1 H)^{4.2563}$ $\quad H = 145{,}448\,(1 - \sigma^{.23496})$

Temperature Ratio $\theta = (1 - A_1 H)$ $\quad H = 145{,}448\,(1 - \theta)$

Altitude H (Ft)	11 To 20 Km 36,089. To 65,617.	20 To 32 Km 65,617. To 104,987.
	$A_2 = 4.80634 \times 10^{-5}$	$A_3 = 3.17175 \times 10^{-6}$
Pressure Ratio	$\delta = .223368\,e^{-A_2(H-36{,}089)}$	$= A_3(1/\theta)^{34.1632}$
Density Ratio	$\sigma = .297088\,e^{-A_2(H-36{,}089)}$	$= A_3(1/\theta)^{35.1632}$
Temp Ratio	$\theta = .75187$	$= .68246 + 1.05778 \times 10^{-6} H$

THERMODYNAMICS

P = Pressure (PSF) $\quad$ V = Volume (Ft^3/Slug) $\quad$ T = Temp (°R)

$PV = RT \qquad P = \rho g R T = 1716\,\rho T$ (Air) $\qquad \rho = 1/V$

Isentropic (Reversible Adiabatic)

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} \qquad V_2 = V_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \qquad T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}$$

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{(\gamma-1)}} \qquad V_2 = V_1\left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} \qquad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

For Air At Atmospheric Temperature Below 50,000 Ft.

$C_p = .240$ BTU/Lb · °F $\qquad C_v = .1715$ BTU/Lb · °F

$$\gamma = C_p/C_v = 1.40 \qquad \rho = .002377\left(\frac{P}{P_o}\right)\left(\frac{T_o}{T}\right)$$

$w = \rho g$ $\quad$ Lbs./Ft^3

Speed of Sound

$a = 49.02\sqrt{T(°R)}$ (Ft/Sec) $\quad = 29.04\sqrt{T(°R)}$ (Knots)

Standard Day Sea Level $\quad a = 1116.45$ Ft/Sec.

BOUNDARY LAYER

Flat Plate $\quad$ Laminar Flow $\quad \delta = \dfrac{5.2\ell}{\sqrt{R_N}}$ $\qquad$ Turbulent Flow $\quad \delta = \dfrac{.37\ell}{R_N^{0.2}}$

$$R_N = \frac{\rho V_T \ell}{\mu} = \frac{V_T \ell}{\nu} \qquad \mu = \frac{2.2697 \times 10^{-8}\,T^{3/2}}{T + 198.72}$$

Standard Day Sea Level $\quad \mu = 3.7372 \times 10^{-7}$ Lb-Sec/Ft^2 $\quad \nu = \mu/\rho$

BASIC LAWS AERODYNAMICS

I Continuity Equation; $\quad \rho AV$ = Constant

II Momentum Equation $\quad F = \dot{m}\,\Delta V$

II Bernoullis Equation (Energy)

Incompressible
$$P_T = P + \frac{\rho V^2}{2}$$

Compressible
$$\frac{\gamma}{\gamma-1}\frac{P_T}{\rho_T} = \frac{\gamma}{\gamma-1}\frac{P}{\rho} + \frac{V^2}{2}$$

Constants

$g = 32.174$ Ft/Sec^2

$R = 53.35$ Ft-Lbs/LB °R Air

$\gamma = 1.4$ Air

Weights	Lbs/Gal	at °C
JP-4	6.55	20
Water	8.345	4

Conversion Factors

bars	x 75.01	= CM Hg 0°c	Ft/Sec	x .6818	= MPH
BTU	x 778.26	= Ft-Lbs	Ft/Sec	x 1.097	= Km/Hr
BTU/Sec	x 1055	= Watts	Ft-Lbs/Sec	x 1.818×10^{-3}	= H.P.
CM Hg	x 27.85	= Lbs/Ft^2	Fluid Oz	x 29.6	= Cu CM
Cu Ft	x 28.32	= Liters	Gals Imp.	x 1.201	= U.S. Gal
dynes	x 2.248×10^{-6}	= Lbs	Kilogram	x 2.205	= Lbs
Ergs	x 7.376×10^{-8}	= Ft-Lbs	Knots	x 1.688	= Ft/Sec
Ft	x .3048	= Meters	Liters	x .2642	= U.S. Gals
Ft/Sec	x .5929	= Kts	Rad	x 57.30	= Deg

AIRSPEED/MACH/q

Dynamic Pressure

$$q = 1/2\,\rho V_T^2 = 1481\,\delta M^2 = \frac{V_e^2}{295} \quad \text{(PSF)}$$

(V_T: Ft/Sec) (V_e: Kts)

Impact Pressure (Pitot-Static Measure) $\quad q_c = (P_T - P_s)$

$$q_c = q F_c \qquad F_c = \left(1 + \frac{M^2}{4} + \frac{M^4}{40} + \frac{M^8}{80}\cdots\right)$$

$$q_c = \left[\left(1 + 0.2M^2\right)^{3.5} - 1\right] P_s \quad \text{(PSF)}$$

Mach No.

$$M = \sqrt{5\left[\left(\frac{P_o}{P}\left\{\left[1 + 0.2\left(\frac{V_c(\text{Kts})}{661.5}\right)^2\right]^{3.5} - 1\right\} + 1\right)^{0.286} - 1\right]}$$

Indicated Airspeed

$$V_I = V_c + \Delta V_{\text{Pitot-Static Source Error}}$$

Calibrated Airspeed

$$V_c = 1479.1\sqrt{\left[1 + \frac{P_T - P_s}{P_o}\right]^{0.28571} - 1} \quad \text{(Knots)}$$

Equivalent Airspeed

$$V_e = 32.174\sqrt{P_s\left\{\left[1 + \frac{P_T - P_s}{P_s}\right]^{0.28571} - 1\right\}} \quad \text{(Knots)}$$

True Airspeed

$$V_T = \frac{V_e}{\sqrt{\sigma}} \qquad M = \frac{V_T}{a}$$

Total Conditions $\quad M < 1.0 \quad \gamma = 1.4$

$$T_T = T\left(1 + \frac{\gamma-1}{2}\epsilon M^2\right) = T\left(1 + .2\,\epsilon M^2\right)$$

$$\rho_T = \rho\left(1 + \frac{\gamma-1}{2}M^2\right)^{1/\gamma-1} = \rho\left(1 + .2M^2\right)^{2.5}$$

$$P_T = P\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} = P\left(1 + .2M^2\right)^{3.5}$$

$M = 1 \quad \gamma = 1.4$

$T/T_T = .833$

$\rho/\rho_T = .634$

$P/P_T = .528$

AERODYNAMIC COEFFICIENTS

$$C_L = \frac{L}{qS} \qquad C_D = \frac{D}{qS} \qquad C_M = \frac{M}{qS\bar{c}} \qquad C_p = \frac{\Delta P}{q}$$

$$C_\ell = \frac{L'}{qSb} \qquad C_N = \frac{N}{qSb} \qquad C_Y = \frac{Y}{qS} \qquad C_T = \frac{T}{qS}$$

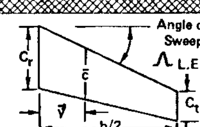
GEOMETRY

For Straight Tapered Surface

Aspect Ratio $\quad A = \dfrac{b^2}{S}$ $\qquad \bar{c} = 2/3\; C_r\left(\dfrac{\lambda^2 + \lambda + 1}{\lambda + 1}\right)$

Taper Ratio $\quad \lambda = C_t/C_r$ $\qquad \bar{y} = \dfrac{b}{6}\left(\dfrac{1 + 2\lambda}{1 + \lambda}\right)$

Area $\quad S = b/2\,(C_t + C_r)$



NOMENCLATURE

A	~	Aspect Ratio
	~	Flow Cross-sectional Area (Ft^2)
a	~	Speed of Sound (Ft/Sec)
	~	Acceleration (Ft/Sec^2)
	~	Lift Curve Slope (1/Deg)
b	~	Wing Span (Ft)
C	~	Coefficient
$\bar{c}$	~	Wing Mean Aerodynamic Chord (Ft)
D	~	Drag (Lbs)
e	~	Wing Efficiency Factor
F	~	Thrust (Lbs)
	~	Force (Lbs)
F_c	~	Compressibility Factor
G(S)	~	Forward Loop Transfer Function
g	~	Acceleration of Gravity (Ft/Sec^2)
H	~	Altitude, Geopotential (Ft)
H(S)	~	Feedback Loop Transfer Function
K	~	Gain
L	~	Lift (Lbs)
Ln	~	Natural Logarithm
L'	~	Rolling Moment (Ft-Lbs)
ℓ	~	Characteristic Length (Ft)
M	~	Pitching Moment (Ft-Lbs)
M	~	Mach
m	~	Mass (Slugs)
N	~	Yawing Moment (Ft-Lbs)
n	~	Load Factor (g's)
P	~	Pressure (PSF)
	~	Roll Rate (Body Axis) Rad/Sec
	~	Time to Oscillate One Cycle (Sec)
Q	~	Pitch Rate (Body Axis) (Rad/Sec)
q	~	Dynamic Pressure (PSF)
R	~	Yaw Rate (Body Axis) Rad/Sec
	~	Gas Constant
	~	Radius (Ft)
R_N	~	Reynolds Number
R/C	~	Rate of Climb (Ft/Min)
S	~	Wing Area (Ft^2)
	~	Takeoff Distance (Ft)
T	~	Thrust (Lbs)
	~	Temperature (°R)
TSFC	~	Thrust Specific Fuel Consumption Lb/Hr/Lb
t	~	Time (Sec)
V	~	Velocity (Ft/Sec)
	~	Volume (Ft^3)
W	~	Gross Weight (Lbs)
W_A	~	Mass Flow Air (Lbs/Sec)
W_f	~	Fuel Flow (Lbs/Hr)
w	~	Specific Weight
Y	~	Side Force (Lbs)

α (alpha)	=	Angle of Attack (Degrees)
β (Beta)	=	Angle of Sideslip
Γ (Gamma)	=	Circulation (Ft^2/Sec)
γ (Gamma)	=	Flight Path Angle (Degrees)
	=	Ratio of Specific Heats, C_p/C_v
Δ (Delta)	=	Increment Notation
δ (Delta)	=	Pressure Ratio, p/p_o
	=	Boundary Layer Thickness (Ft)
	=	Control Surface Deflection (Deg)
ϵ (Epsilon)	=	Temperature Probe Recovery Factor
	=	Atmospheric Eddy Viscosity
	=	Downwash Angle (Deg)
ζ (Zeta)	=	Damping Parameter
θ (Theta)	=	Temperature Ratio, T/T_o
	=	Pitch Attitude (Earth Axis System)
Λ (Lambda)	=	Sweep Angle (Deg)
λ (Lambda)	=	Taper Ratio
μ (Mu)	=	Coefficient of Tire Friction
	=	Viscosity Lb · Sec/Ft^2
ν (Nu)	=	Kinematic Viscosity (Ft^2/Sec)
ρ (Rho)	=	Mass Air Density (Slugs/Ft^3)
σ (Sigma)	=	Density Ratio, ρ/ρ_o
	=	Sidewash Angle (Deg)
τ (Tau)	=	Time Constant (Sec)
ϕ (Phi)	=	Bank Angle (Earth Axis System) (Deg)
	=	Runway Gradient (Deg)
ψ (Psi)	=	Yaw Angle (Earth Axis System) (Deg)
ω (Omega)	=	Frequency (Rad/Sec)
$(\dot{X})$	=	Dot over Parameter d(X)/dt
$(\bar{X})$	=	Bar over Parameter Average Value

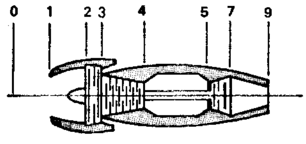
Subscripts

A	~ Aileron	ℓ	~ Roll
a.c.	~ Aerodynamic Center	m	~ Pitch
am	~ Ambient	n	~ Yaw
C	~ Calibrated		~ Net
	~ Compressible		~ Natural
	~ Chord	o	~ Sea Level
CG	~ Center of Gravity	P	~ Pressure
D	~ Drag	R	~ Rudder
Dp	~ Parasite Drag	S	~ Static
d	~ Damping	T	~ Total
E	~ Elevator		~ True
e	~ Equivalent		~ Tropopause Conditions
f	~ Skin Friction		~ Thrust
G	~ Gross	t	~ Tail
	~ Ground	W	~ Wind
I	~ Indicated		~ Wing
i	~ Incidence	Y	~ Side Force
L	~ Lift		

JET ENGINE PERFORMANCE

Gross Thrust

P & W Engine Stations (J-57 and TF33)



$$F_G = \frac{W_a}{g} V_9 + A_9 (P_9 - P_o)$$

$$F_n = \frac{W_a}{g}(V_9 - V_o) + A_9 (P_9 - P_o)$$

$$\frac{F_n}{\delta_{am}} = \underbrace{\underbrace{P_o \psi_f C_f A_f}_{\text{Fan}} + \underbrace{P_o \psi_p C_g A_c \left(\frac{A_h}{A_c}\right)}_{\text{Primary}}}_{\text{Gross Thrust}} - \underbrace{\left(\frac{F_r/\delta_{t_2}}{\frac{W_a\sqrt{\theta_{t_2}}}{\delta_{t_2}}}\right)\left(\frac{W_a\sqrt{\theta_{t_2}}}{\delta_{t_2}}\right)\left(\frac{P_{t_2}}{P_{t_1}}\right)\left(\frac{P_{t_1}}{P_{am}}\right)}_{\text{Ram Drag}}$$

$$\psi_{unchoked} = \frac{2\gamma}{\gamma-1}\left[\left(\frac{P_t}{P_{am}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad \psi_{choked} = 2\,\frac{P_t}{P_{am}}\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} - 1$$

C_f, C_g ~ Gross Thrust Coefficient Fan $f(P_{T_3}/P_{am})$, Primary $f(P_{T_7}/P_{am})$

A_f, A_c ~ Exhaust Exit Area Fan, Primary

A_h/A_c ~ Ratio of A_9 Hot To Cold $f(T_{t_7})$

$\frac{W_a\sqrt{\theta_{t_2}}}{\delta_{t_2}}$ is corrected engine airflow, Lbs/Sec $f\left(P_{t_2}/P_{am},\ \frac{N}{\sqrt{\theta_{t_2}}}\right)$

$\frac{F_r/\delta_{t_2}}{W_a\sqrt{\theta_{t_2}}/\delta_{t_2}}$ is ram drag parameter, $\frac{\text{Lbs}}{\text{Lbs/Sec}} = \frac{a_o}{g}\,\frac{M}{\sqrt{1+\frac{\gamma-1}{2}M^2}}$

$\frac{P_{t_2}}{P_{t_1}}$ is inlet pressure recovery ratio, dimensionless $f\left(W_a\sqrt{\theta_{t_2}}/\delta_{t_2},\ M\right)$

$\frac{P_{t_1}}{P_{am}}$ is ram pressure ratio, dimensionless $= \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$

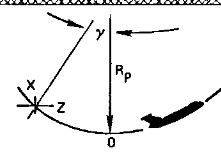
MANEUVERS

Steady State Pullup

$$Z = R_P - \sqrt{R_P^2 - X^2}$$

$$R_P = V^2/11.29(n - \cos\gamma)$$
(Ft) (Kts)

$$Q = \dot{\gamma} = \frac{g(n-\cos\gamma)}{V}$$
(Rad/Sec)



Steady State Turn

$$n = 1/\cos\phi$$

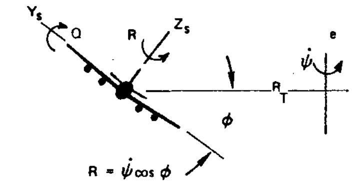
$$\dot{\psi} = \frac{g\tan\phi}{V} \sim \left(\frac{\text{Rad}}{\text{Sec}}\right)$$

$$R_T = V^2/2132.5\, g\tan\phi$$
(N.M.) (Kts)

$$Q = \dot{\psi}\sin\phi \qquad R = \dot{\psi}\cos\phi$$

Stab Axis $Q = \frac{g}{V}\left(n - \frac{1}{n}\right)$, $R = \frac{g}{nV}\sqrt{n^2-1}$

(Rad/Sec) (Rad/Sec)

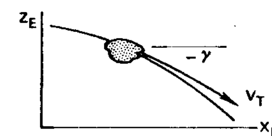


Time 360° ψ = .0055 V/tan ϕ (Minutes)
(KTS)

TRAJECTORY

$$\frac{dV}{dt} = -g\left(\sin\gamma + \frac{C_D S q}{W}\right)$$

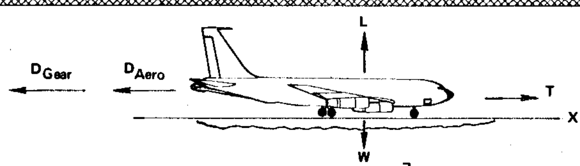
$$\frac{d\gamma}{dt} = \frac{-g\cos\gamma}{V}$$



$$V_{T_X} = V_T\cos\gamma \qquad V_{T_Z} = V_T\sin\gamma$$

JET AIRPLANE PERFORMANCE

Ground



$$a_x = \frac{g}{W}\left[T_T - \mu W - (C_D - \mu C_L)\, qS - W\sin\phi\right]$$

For Average Values T_T, W, C_D, C_L, μ, ϕ (Runway Gradient) At .707 $V_{T.O.}$

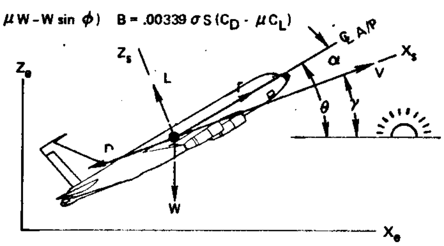
$$S = \frac{.0444W}{B}\,\text{Ln}\left[\frac{A - BV_W^2}{A - BV_{T.O.}^2}\right] - 1.69\, V_{Wt} \qquad t = \frac{.026W}{\sqrt{AB}}\,\text{Ln}\left\{\left[\frac{1+\sqrt{B/A}\,V_{T.O.}}{1-\sqrt{B/A}\,V_{T.O.}}\right]\left[\frac{1-\sqrt{B/A}\,V_W}{1+\sqrt{B/A}\,V_W}\right]\right\}$$

($V_{T.O.}$ ~ KTAS) (V_W ~ Kts) $A = (T - \mu W - W\sin\phi)$ $B = .00339\,\sigma S(C_D - \mu C_L)$

In Flight

$\dot{u}$ ~ Long Accel X_s Axis

$\dot{w}$ ~ Vert. Accel Z_s Axis



Wings Level $\phi = 0$

$$\dot{u} = \frac{g}{W}\left[T\cos\alpha - D - W\sin\gamma\right] \qquad n_Z = 1 - \left[\frac{\dot{w}}{g} - \frac{\dot{\theta}V}{g}\right]$$

$$\dot{w} = \frac{g}{W}\left[L - W\cos\gamma + T\sin\alpha\right] + V\dot{\theta}$$

Energy Equation T.E. = P.E. + K.E. = W·h + 1/2 m V_T^2

Specific Energy $E_h = h + \frac{V^2}{2g}$ $\frac{dE_h}{dt} = \frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt} = \frac{(T-D)V}{W}$

T.E. Total Energy P.E. Potential Energy K.E. Kinetic Energy

Climb: Descent

$$R/C = 101.28\left[\frac{\left(\frac{T-D}{W}\right)V_T\ (\text{Kts})}{1+\left(\frac{V}{g}\frac{dV}{dh}\right)}\right] \text{ Ft/Min}$$

$\left(\frac{V}{g}\frac{dV}{dh}\right)$ For Constant	V_c	V_e	M
Above 36,089	–	$.7M^2$	0
Below 36,089	▷	$.567M^2$	$-.133M^2$

▷ $\frac{V}{g}\left(\frac{dh}{dV}\right) = -.014055M + .656671M^2 - .214204M^3$

Range

$$\underset{(N.M.)}{R} = \frac{a_o\sqrt{\theta}}{TSFC}\left(M\frac{L}{D}\right)\text{Ln}\left(\frac{W_1}{W_2}\right) = W\,\frac{NAM}{Lb}\,\text{Ln}\left(\frac{W_1}{W_2}\right)$$

$W\,\frac{NAM}{Lb}$ ~ Range Factor

Endurance

$$\underset{(Hrs)}{T} = \frac{1}{TSFC}\left(\frac{L}{D}\right)\text{Ln}\left(\frac{W_1}{W_2}\right) = \frac{W}{W_f}\,\text{Ln}\left(\frac{W_1}{W_2}\right)$$

$\frac{NAM}{Lb}$ ~ Nautical Air Miles Per Lb Fuel

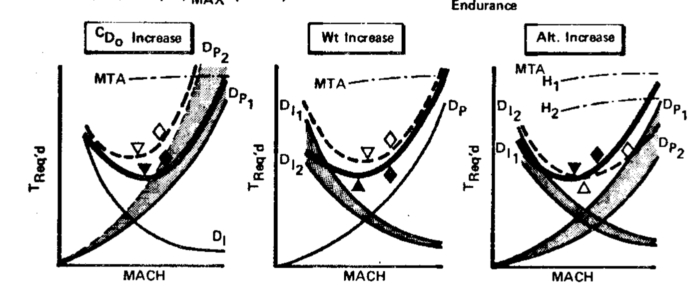
a_o Speed Sound Knots At Sea Level

Parabolic Drag Polar, TSFC ≠ f (M)

$$\underset{(N.M.)}{R_{MAX}} = \frac{34.40}{TSFC\sqrt{\sigma S}}\left(\frac{C_L^{1/2}}{C_D}\right)_{MAX}\left(\sqrt{W_1} - \sqrt{W_2}\right)$$

$$C_D = C_{D_o} + \frac{C_L^2}{\pi e A} \qquad \underset{\text{Range}}{C_{L_{MAX}}} = \sqrt{\pi A e C_{D_o}/3} \qquad \left(\frac{C_L^{1/2}}{C_D}\right)_{MAX} = 3\sqrt{C_{L_{MAX\ Range}}}\,/\,4C_{D_o}$$

$$\underset{(Hrs)}{T_{MAX}} = \left(\frac{L}{D}\right)_{MAX}\left(\frac{1}{TSFC}\right)\text{Ln}\,\frac{W_1}{W_2} \qquad \underset{\text{Endurance}}{C_{L_{MAX}}} = \sqrt{\pi A e C_{D_o}}$$



Baseline ▬▬ Optimum Range ◆ Endurance ▼ D_I ~ Induced Drag Contribution MTA ~ Maximum
Increase – – – Optimum Range ◇ Endurance ▽ D_P ~ Parasite Drag Contribution Thrust Available

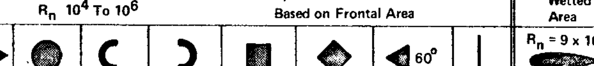
Range Correction Factor

$$\left[1 + \frac{\Delta R}{R}\right] = \frac{1}{\left[1+\frac{\Delta C_D}{C_D}\right]\left[1+\frac{\Delta TSFC}{TSFC}\right]}$$

Drag Coefficient Data

One Side Flat Plate: Laminar Flow $C_f = \frac{1.328}{\sqrt{R_n}}$ | Turbulent Flow $C_f = \frac{0.455}{(\text{Log}_{10} R_n)^{2.58}}$

C_D R_n 10^4 To 10^6 Based on Frontal Area — Wetted Area



Direction of Flow →						60°		$R_n = 9\times10^6$, t/c = 12%
2-Dim.	1.17	1.20	2.30	2.05	1.55	1.55	1.98	.0058
3-Dim.	.47	.38	1.42	1.05	.80	.50	1.17	.0030

NAVIGATION

Straight Line Navigation (Rhumbline Navigation)

Two Points A and B Lat ~ Latitude (Deg, + West)

Long ~ Longitude (Deg + Northern)

$$\underset{(\text{Deg North})}{\psi} = \tan^{-1}\left[\frac{\pi\,(\text{Long}_A - \text{Long}_B)}{180\left[\text{Ln}\tan(45° + .5\,\text{Lat}_B) - \text{Ln}\tan(45° + .5\,\text{Lat}_A)\right]}\right]$$

If cos ψ = 0

$$\underset{(N.M.)}{S} = 60\,(\text{Lat}_B - \text{Lat}_A)/\cos\psi = 60\,(\text{Long}_B - \text{Long}_A)\cos(\text{Lat})$$

Great Circle Navigation

$$\underset{(N.M.)}{S} = 60\cos^{-1}\left[\sin\text{Lat}_A\sin\text{Lat}_B + \cos\text{Lat}_A\cos\text{Lat}_B\cos(\text{Long}_B - \text{Long}_A)\right]$$

AERODYNAMIC SUMMARY SHEET 2

LONGITUDINAL STABILITY

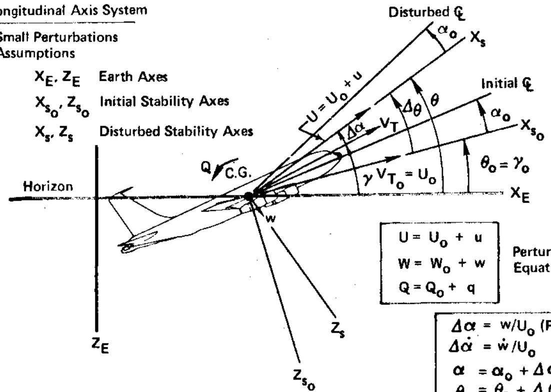
Longitudinal Axis System

Small Perturbations Assumptions

X_E, Z_E Earth Axes

X_{s_o}, Z_{s_o} Initial Stability Axes

X_s, Z_s Disturbed Stability Axes



Perturbation Equations

$$U = U_o + u$$
$$W = W_o + w$$
$$Q = Q_o + q$$

$$\Delta\alpha = w/U_o \text{ (Rad)}$$
$$\Delta\dot{\alpha} = \dot{w}/U_o$$
$$\alpha = \alpha_o + \Delta\alpha$$
$$\theta = \theta_o + \Delta\theta$$
$$\gamma = \theta - \Delta\alpha$$

Longitudinal Equations of Motion

$$\dot{u} = -g\theta \cos \theta_o + X_u u + X_{T_u} u + X_\alpha \Delta\alpha + X_{\delta_E} \delta_E \quad \text{Ft/Sec}^2$$

$$\dot{w} - U_o q = -g\theta \sin \theta_o + Z_u u + Z_\alpha \Delta\alpha + Z_{\dot{\alpha}} \Delta\dot{\alpha} + Z_q q + Z_{\delta_E} \delta_E \quad \text{Ft/Sec}^2$$

$$\dot{q} = M_u u + M_{T_u} u + M_\alpha \Delta\alpha + M_{T_\alpha} \alpha + M_{\dot{\alpha}} \Delta\dot{\alpha} + M_q q + M_{\delta_E} \delta_E \quad \text{Rad/Sec}^2$$

Longitudinal Dimensional Stability Derivatives

$$X_u = \frac{-q S(C_{D_u} + 2C_{D_o})}{mU_o} \text{ (Sec}^{-1}\text{)}$$

$$X_{T_u} = \frac{q S(C_{T_{x_u}} + 2C_{T_x})}{mU_o} \text{ (Sec}^{-1}\text{)}$$

$$X_\alpha = \frac{-q S(C_{D_\alpha} - C_{L_o})}{m} \text{ (Ft Sec}^{-2}\text{)}$$

$$X_{\delta_E} = \frac{-q SC_{D\delta_E}}{m} \text{ (Ft Sec}^{-2}\text{Deg}^{-1}\text{)}$$

$$Z_u = \frac{-q S(C_{L_u} + 2C_{L_o})}{mU_o} \text{ (Sec}^{-1}\text{)}$$

$$Z_\alpha = \frac{-q S(C_{L_\alpha} + C_{D_o})}{m} \text{ (Ft Sec}^{-2}\text{)}$$

$$Z_{\dot{\alpha}} = \frac{-q SC_{L_{\dot{\alpha}}} \bar{c}}{2mU_o} \text{ (Ft Sec}^{-1}\text{)}$$

$$Z_q = \frac{-\bar{q} SC_{L_q} \bar{c}}{2mU_o} \text{ (Ft Sec}^{-1}\text{)}$$

$$Z_{\delta_E} = \frac{-q SC_{L_{\delta_E}}}{m} \text{ (Ft Sec}^{-2}\text{ Deg}^{-1}\text{)}$$

$$M_u = \frac{q S\bar{c}(C_{m_u} + 2C_{m_o})}{I_{yy}U_o} \text{ (Ft}^{-1}\text{ Sec}^{-1}\text{)}$$

$$M_{T_u} = \frac{q S\bar{c}(C_{m_{T_u}} + 2C_{m_T})}{I_{yy}U_o} \text{ (Ft}^{-1}\text{ Sec}^{-1}\text{)}$$

$$M_\alpha = \frac{q S\bar{c}C_{m_\alpha}}{I_{yy}} \text{ (Sec}^{-2}\text{)}$$

$$M_{T_\alpha} = \frac{q S\bar{c}C_{m_{T_\alpha}}}{I_{yy}} \text{ (Sec}^{-2}\text{)}$$

$$M_{\dot{\alpha}} = \frac{q S\bar{c}^2 C_{m_{\dot{\alpha}}}}{2 I_{yy}U_o} \text{ (Sec}^{-1}\text{)}$$

$$M_q = \frac{q S\bar{c}^2 C_{m_q}}{2 I_{yy}U_o} \text{ (Sec}^{-1}\text{)}$$

$$M_{\delta_E} = \frac{q S\bar{c}C_{m_{\delta_E}}}{I_{yy}} \text{ (Sec}^{-2}\text{Deg}^{-1}\text{)}$$

LATERAL DIRECTIONAL STABILITY

Lateral - Directional Axis System

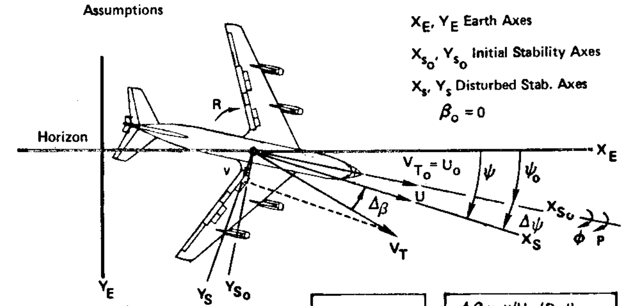
Small Perturbations Assumptions

X_E, Y_E Earth Axes

X_{s_o}, Y_{s_o} Initial Stability Axes

X_s, Y_s Disturbed Stab. Axes

$\beta_o = 0$



Perturbation Equations

$$V = V_o + v$$
$$P = P_o + p$$
$$R = R_o + r$$

$$\Delta\beta = v/U_o \text{ (Rad)}$$
$$\Delta\dot{\beta} = \dot{v}/U_o \text{ (Rad/Sec)}$$
$$\psi = \psi_o + \Delta\psi$$
$$\phi = \Delta\phi$$

Lateral — Directional Equations of Motion

$$\dot{v} + U_o r = g\phi \cos\theta_o + Y_\beta \Delta\beta + Y_p p + Y_r r + Y_{\delta_A} \delta_A + Y_{\delta_R} \delta_R \quad \text{Ft/Sec}^2$$

$$\dot{p} - (I_{xz}/I_{xx}) \dot{r} = L_\beta \Delta\beta + L_p p + L_r r + L_{\delta_A} \delta_A + L_{\delta_R} \delta_R \quad \text{Rad/Sec}^2$$

$$\dot{r} - (I_{xz}/I_{zz}) \dot{p} = N_\beta \Delta\beta + N_{T_\beta} \Delta\beta + N_p p + N_r r + N_{\delta_A} \delta_A + N_{\delta_R} \delta_R \quad \text{Rad/Sec}^2$$

Lateral-Directional Dimensional Stability Derivatives

$$Y_\beta = \frac{q SC_{y_\beta}}{m} \text{ (Ft Sec}^{-2}\text{)}$$

$$Y_p = \frac{q SbC_{y_p}}{2mU_o} \text{ (Ft Sec}^{-1}\text{)}$$

$$Y_r = \frac{q SbC_{y_r}}{2mU_o} \text{ (Ft Sec}^{-1}\text{)}$$

$$Y_{\delta_A} = \frac{q SC_{y\delta_A}}{m} \text{ (Ft Sec}^{-2}\text{Deg}^{-1}\text{)}$$

$$Y_{\delta_R} = \frac{q SC_{y\delta_R}}{m} \text{ (Ft Sec}^{-2}\text{Deg}^{-1}\text{)}$$

$$L_\beta = \frac{q SbC_{\ell_\beta}}{I_{xx}} \text{ (Sec}^{-2}\text{)}$$

$$L_p = \frac{q Sb^2 C_{\ell_p}}{2 I_{xx}U_o} \text{ (Sec}^{-1}\text{)}$$

$$L_r = \frac{q Sb^2 C_{\ell_r}}{2 I_{xx}U_o} \text{ (Sec}^{-1}\text{)}$$

$$L_{\delta_A} = \frac{q SbC_{\ell_{\delta_A}}}{I_{xx}} \text{ (Sec}^{-2}\text{ Deg}^{-1}\text{)}$$

$$L_{\delta_R} = \frac{q SbC_{\ell_{\delta_R}}}{I_{xx}} \text{ (Sec}^{-2}\text{ Deg}^{-1}\text{)}$$

$$N_\beta = \frac{q SbC_{n_\beta}}{I_{zz}} \text{ (Sec}^{-2}\text{)}$$

$$N_{T_\beta} = \frac{q SbC_{n_{T_\beta}}}{I_{zz}} \text{ (Sec}^{-2}\text{)}$$

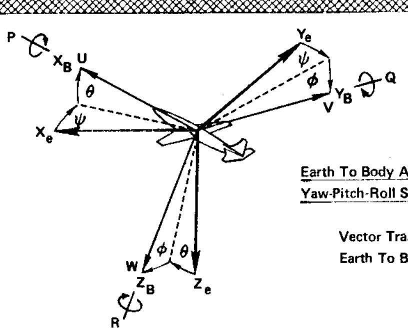
$$N_p = \frac{q Sb^2 C_{n_p}}{2 I_{zz}U_o} \text{ (Sec}^{-1}\text{)}$$

$$N_r = \frac{q Sb^2 C_{n_r}}{2 I_{zz}U_o} \text{ (Sec}^{-1}\text{)}$$

$$N_{\delta_A} = \frac{q SbC_{n_{\delta_A}}}{I_{zz}} \text{ (Sec}^{-2}\text{ Deg}^{-1}\text{)}$$

$$N_{\delta_R} = \frac{q SbC_{n_{\delta_R}}}{I_{zz}} \text{ (Sec}^{-2}\text{ Deg}^{-1}\text{)}$$

AXIS TRANSFORMATION



Earth To Body Axis
Yaw-Pitch-Roll Sequence

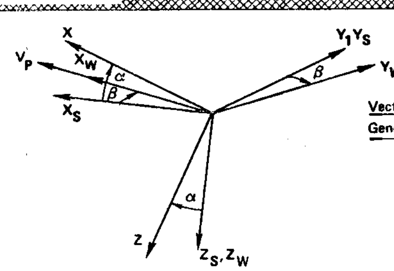
Vector Transformation Matrix
Earth To Body Axes (Yaw-Pitch-Roll Sequence)

Body Axes Components	Components Along Earth Axes: X_e	Y_e	Z_e
X	$\cos\theta \cos\psi$	$\cos\theta \sin\psi$	$-\sin\theta$
Y	$\sin\psi \sin\theta \cos\psi$ $-\cos\phi \sin\psi$	$\sin\phi \sin\theta \sin\psi$ $+\cos\phi \cos\psi$	$\sin\phi \cos\theta$
Z	$\cos\phi \sin\theta \cos\psi$ $+\sin\phi \sin\psi$	$\cos\phi \sin\theta \sin\psi$ $-\sin\phi \cos\psi$	$\cos\phi \cos\theta$

Angular-Velocity Relations

$P = \dot{\phi} - \dot{\psi} \sin\Theta$	$\dot{\psi} = Q \sin\phi \sec\Theta + R \cos\phi \sec\theta$
$Q = \dot{\Theta} \cos\phi + \dot{\psi} \sin\phi \cos\theta$	$\dot{\theta} = Q \cos\phi - R \sin\phi$
$R = -\dot{\Theta} \sin\phi + \dot{\psi} \cos\phi \cos\Theta$	$\dot{\phi} = P + Q \sin\phi \tan\Theta + R \cos\phi \tan\theta$

Note: Vectors X, Y & Z can be acceleration, velocity or distance



Vector Transformation Matrix
General Wind to Body Axes

Body, Stability, and Wind-Axes
Yaw-Pitch Sequence

[Yaw-Pitch Sequence
+ β Relative Wind Right of Airplane ℄]

Body Axis Comp.	Components Along Wind Axes: X.	Y.	Z.
X	$\cos\alpha \cos\beta$	$-\cos\alpha \sin\beta$	$-\sin\alpha$
Y	$\sin\beta$	$\cos\beta$	0
Z	$\sin\alpha \cos\beta$	$-\sin\alpha \sin\beta$	$\cos\alpha$

Angular-Velocity Relations

$P = \dot{\beta} \sin\alpha$	$\dot{\beta} = -R \sec\alpha = P \csc\alpha$
$Q = \dot{\alpha}$	$\dot{\alpha} = Q$
$R = -\dot{\beta} \cos\alpha$	$0 = P + R \tan\alpha$

Body Axis To Stability Axis
Anglular-Velocity Relations

$P_s = P \cos\alpha + R \sin\alpha$	$P = P_s \cos\alpha - R_s \sin\alpha$
$Q_s = Q$	$Q = Q_s$
$R_s = -P \sin\alpha + R \cos\alpha$	$R = P_s \sin\alpha + R_s \cos\alpha$

LONGITUDINAL

Static Trim

$$C_{M_{CG}} = C_{M_{a.c.\,WB}} + C_L \frac{X}{\bar{c}} + C_D \frac{Z}{\bar{c}} + C_{M_{NAC}} - C_{L_t} \frac{q_t}{q} \bar{V}_H$$

$$C_{L_t} = a_t \left(\alpha_W - \alpha_{W_i} + \alpha_{t_i} - \epsilon\right) \quad \text{For Symmetrical Airfoil}$$

Static Stability

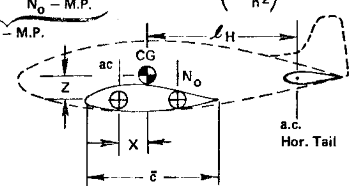
$$C_{M\alpha} = C_{L_\alpha} \frac{X}{\bar{c}} + \frac{dC_D}{d\alpha}\frac{Z}{\bar{c}} + \frac{dC_{M_{NAC}}}{d\alpha} - C_{L_{\alpha H}} \frac{q_t}{q} \bar{V}_H \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

Elevator/g

$$\frac{d\,\delta_e}{d\,n_z} = \frac{W/S}{qC_{M\,\delta_e}} \underbrace{\left[\left(\frac{dC_m}{dC_L}\right) + \underbrace{\frac{C_{m_q}\bar{c}\rho g A}{4W/S}}_{N_o - M.P.}\right]}_{CG - M.P.}$$

$A = 1$ For Pullup

$A = \left(1 + \frac{1}{n^2}\right)$ Turn



Neutral Point (N_o)

$$N_o = \bar{X}_{CG} - \left(\frac{dC_M}{dC_L}\right)_{Airplane}$$

$$C_{M\alpha} = C_{L\alpha}\left(\bar{X}_{CG} - N_o\right)$$

$$N_o = N_{o_{Wing\,Body}} - \frac{dC_M}{dC_{L\,Nac.}} + \frac{a_t}{a_w}\left(\frac{q_t}{q}\right)\bar{V}_H\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

$$\bar{V}_H = S_H\, \ell_H / S\bar{c}$$

Maneuver Point (M.P.)

$$M.P. = \overbrace{N_o - \frac{C_{M_q}\rho S\bar{c}g}{4W}}^{\text{Pull Up}} = \overbrace{N_o - \frac{C_{M_q}\rho S\bar{c}g}{4W}\left(1 + \frac{1}{n^2}\right)}^{\text{Turn}}$$

Short Period Approximation

Natural Freq. $\quad \omega_n = \sqrt{\frac{Z_\alpha M_q}{U_o} - M_\alpha}$ Rad/Sec

Damping $\quad \zeta = \frac{-\left(M_q + \frac{Z_\alpha}{U_o} + M_{\dot\alpha}\right)}{2\,\omega_{n_{sp}}}$

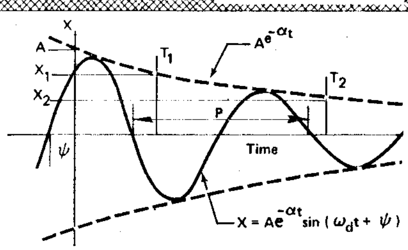
Phugoid Approximation

$$\omega_n = \sqrt{\frac{-Z_U g}{U_o}} \text{ Rad/Sec} \qquad \zeta = \frac{-X_U}{2\,\omega_{n_p}}$$

HINGE MOMENTS

$$C_H = \frac{HM}{qS_c\bar{c}_c} \qquad C_H = C_{H_o} + C_{H_{\delta_{TAB}}}\delta_{TAB} + C_{H_{\delta_c}}\delta_c + C_{H_\alpha}\alpha$$

OSCILLATORY MOTION



$$\omega_d = 2\pi / P_d$$

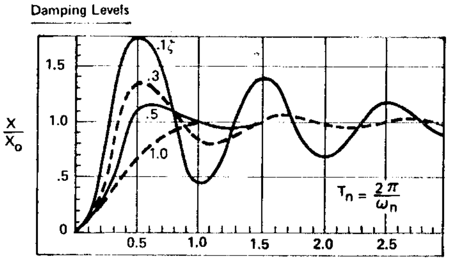
$$\sigma = \zeta\,\omega_n$$

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

$$\sigma = \frac{\text{Ln}\left(\frac{X_1}{X_2}\right)}{T_2 - T_1}$$

$$T_{1/2\,Amp} = \frac{.693}{\zeta\omega_n}$$

Damping Levels



Root Locus

$\zeta = \text{Sin}\,\theta$

$\omega_d = \omega_n\ \text{Cos}\,\theta$

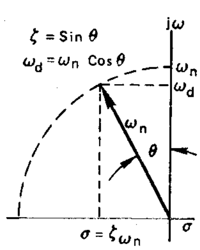


TABLE OF LAPLACE TRANSFORMS

Time Function		Laplace Transform
Unit Impulse	$\delta(t)$	1
Unit Step	$u(t)$	$1/s$
Unit Ramp	t	$1/s^2$
Polynomial	t^n	$n!/s^{n+1}$
Exponential	e^{-at}	$\frac{1}{s+a}$
Sine Wave	$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
Cosine Wave	$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
Damped Sine Wave	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
Damped Cosine Wave	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

LATERAL-DIRECTIONAL

Static Trim

$$C_{\ell_\beta}\beta + C_{\ell_{\delta_A}}\delta_A + C_{\ell_{\delta_R}}\delta_R + L'_T/qSb = 0$$

$$C_{n_\beta}\beta + C_{n_{\delta_A}}\delta_A + C_{n_{\delta_R}}\delta_R + N_T/qSb = 0$$

$$C_{Y_\beta}\beta + C_{Y_{\delta_A}}\delta_A + C_{Y_{\delta_R}}\delta_R + C_L\sin\phi\,\cos\gamma = 0$$

Static Stability

$$C_{n_\beta} = -a_v\left(\frac{q_t}{q}\right)\bar{V}_v\left(1 - \frac{d\sigma}{d\beta}\right)$$

$$\bar{V}_v = S_v\ell_v/Sb$$

Roll Performance

$$\phi(t) = -\frac{L_{\delta_A}\delta_A}{L_p}\,t + \frac{L_{\delta_A}\delta_A}{L_p^2}\left(e^{L_p t} - 1\right)$$

$$P(t) = P_{ss}\left(1 - e^{-t/T_R}\right)$$

$$P_{ss} = -\frac{L_{\delta_A}\delta_A}{L_p} \qquad \frac{Pb}{2V} = \frac{C_{\ell_{\delta_A}}}{C_{\ell_p}}\delta_A \quad \text{Helix Angle} \quad \text{Rad.}$$

Roll Time Contant

$$\frac{1}{T_R} = -\frac{\rho S U_o b^2}{4\,I_{xx}}\left[C_{\ell_p} + \frac{C_{\ell_\beta}}{C_{n_\beta}}\left(\frac{2\,I_{zz}}{m_b^2}C_L - C_{n_p}\right)\right] \quad 1/\text{Sec}$$

Spiral Time Constant

$$\frac{1}{T_S} = -\frac{g}{U_o C_{\ell_p}}\left(\frac{C_{\ell_\beta}}{C_{n_\beta}}C_{n_r} - C_{\ell_r}\right) \quad 1/\text{Sec}$$

Dutch Roll Approximation

$$\omega_n = \sqrt{\frac{1}{U_o}\left(Y_\beta N_r + N_\beta U_o - N_\beta Y_R\right)} \quad \text{Rad/Sec}$$

$$\zeta_d = -\frac{\rho S U_o}{4\omega_n m}\left\{C_{Y_\beta} + \frac{m\,b^2}{2\,I_{zz}}C_{n_r} - \left(\frac{I_{zz}}{I_{xx}}\frac{C_{\ell_\beta}}{C_{n_\beta}}\right)\left(C_L - \frac{mb^2}{2\,I_{zz}}C_{n_p}\right)\right\}$$

$$\left|\frac{\phi}{\beta}\right|_d = -\frac{C_{\ell_\beta}\,I_{zz}}{C_{n_\beta}\,I_{xx}\sqrt{1 + \frac{1}{\omega_d^2\,T_R^2}}}$$

STABILITY DERIVATIVE EFFECT

	Stability Derivative	Quantity Most Affected	How Affected
Longitudinal	C_{m_q}	Damping of the short period	Increase C_{m_q} to increase the damping
	C_{m_α}	Natural frequency of the short period	Increase C_{m_α} to increase the frequency
	C_{x_u}	Damping of the phugoid	Increase C_{x_u} to increase the damping
	C_{z_u}	Natural frequency of the phugoid	Increase C_{z_u} to increase the frequency
Lateral Directional	C_{n_r}	Damping of the Dutch roll	Increase C_{n_r} to increase the damping
	C_{n_β}	Natural frequency of the Dutch roll	Increase C_{n_β} to increase the natural frequency
	C_{ℓ_p}	Roll subsidence	Increase C_{ℓ_p} to increase $1/T_R$
	C_{ℓ_β}	Spiral divergence	Increase C_{ℓ_β} for spiral stability

FEEDBACK CONTROL SYSTEMS

Transfer Function

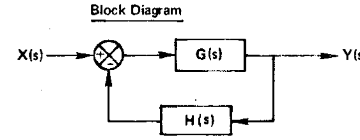
Closed-Loop

$$\frac{Y(s)}{X(s)} = \left[\frac{G(s)}{1 + G(s)\,H(s)}\right]$$

Open-Loop

$G(s)\ H(s)$

Block Diagram



Root-Locus Analysis

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1 + K\frac{N(s)}{D(s)}} \qquad K\frac{N(s)}{D(s)} = \frac{K(s + Z_1)(s + Z_1)\cdots(s + Z_m)}{(s + P_1)(s + P_2)\cdots(s + P_n)}$$

$m \le n$

$s \sim$ Laplace Operator

(o) Zeros $\sim$ Roots of Numerator $Z_1, Z_2 \cdots \cdots\cdots Z_m$

(x) Poles $\sim$ Roots of Denominator $P_1, P_2 \cdots \cdots P_n$

Bode Diagram ~ Steady State Response To Sinusoidal Input

Bode Form Transfer Function

$$P(j\omega) = \frac{K_B\left(1 + \frac{j\omega}{Z_1}\right)\left(1 + \frac{j\omega}{Z_2}\right)\cdots\left(1 + \frac{j\omega}{Z_m}\right)}{(j\omega)^\ell\left(1 + \frac{j\omega}{P_1}\right)\left(1 + \frac{j\omega}{P_2}\right)\cdots\left(1 + \frac{j\omega}{P_n}\right)}$$

$$K_B = K\left[\sum_{i=1}^{m} Z_i \Big/ \sum_{i=1}^{n} P_i\right]$$

Amplitude

$$db = 20\,\text{Log}\left|P(j\omega)\right|$$

Break Point

$$\omega = \frac{1}{P_i}$$

Phase Angle

$$\phi = \tan^{-1}\frac{\omega}{Z_m} \qquad \omega \ll P,\ \phi = 0^\circ \qquad \omega = P,\ \phi = 45^\circ$$

$$\phi = -\tan^{-1}\frac{\omega}{P} \qquad \omega \gg P,\ \phi = 90^\circ\ \text{(Lag)}$$